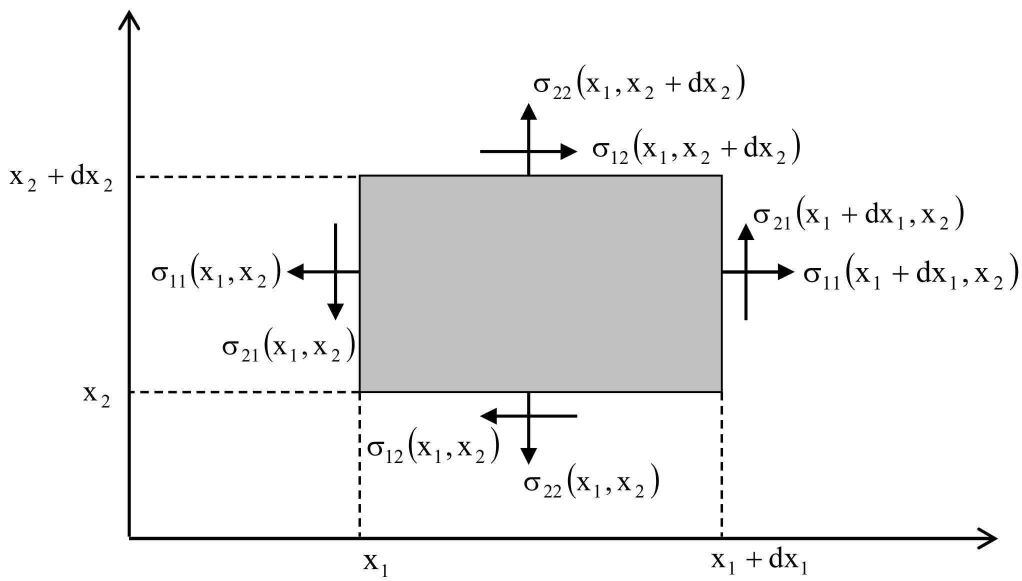
Exercise 1: The Cauchy stress tensor at a given point P of a solid is given by the following matrix,

$$[\sigma] = \begin{pmatrix} \sigma_{11} & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

Determine σ_{11} so that there is a plane on which the stress vector is zero. Then find the orientation of this plane with respect to the coordinate system describing the given stress sate.

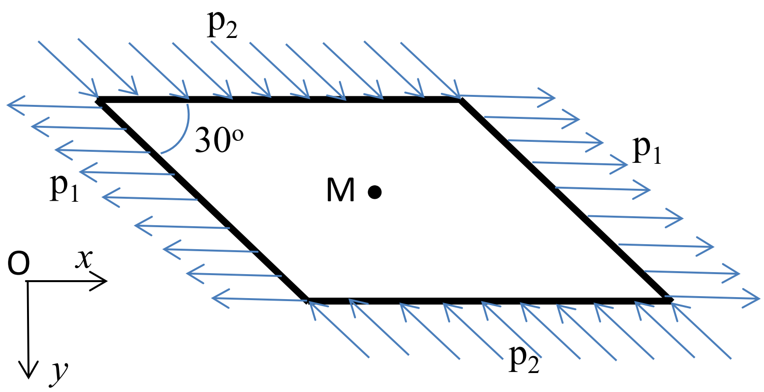
Exercise 2: Consider an element of a thin plate (taken as a unit thickness) as shown in the Figure. The element is in the state of plane stress, i.e. $\sigma_{33} = \sigma_{13} = \sigma_{31} = \sigma_{23} = \sigma_{32} = 0$. Write down the equations of equilibrium and show that,

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} = 0, \quad \frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} = 0, \quad \sigma_{21} = \sigma_{12}$$



Exercise 3: The Cauchy stress field in a solid is given by, $\sigma_{ij} = \dfrac{a x_i x_j x_3}{R^5}$ where $R^2 = x_1^2 + x_2^2 + x_3^2$ and $a =$constant. Examine if there are body forces acting on the body.

Exercise 4: A thin plate in a form of a parallelogram on the plane O*xy* is subjected to uniform stresses p_1 = 150 MPa et p_2 =70 MPa, as shown in the figure. Calculate (a) the stresses: σ_x, σ_y, $\tau(\tau_{xy} = \tau_{yx})$ on the two planes normal to *x* and *y*, (b) the principal stresses and their orientation, (c) draw the corresponding Mohr's circle and evaluate the principal stresses as well as their directions.



Exercise 5: Show that on the octahedral plane the normal and shear stresses are given by the following expressions.

$$t_N = I_1(\sigma)/3, \quad t_T = \frac{1}{3}\sqrt{2I_1^2(\sigma) - 6I_2(\sigma)}, \quad t_T = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

Exercise 6: The state of stress at a point P relative to a coordinate system, x_1, x_2, x_3 is (units in MPa),

$$[\sigma] = \begin{pmatrix} 20 & 10 & -10 \\ 10 & 30 & 0 \\ -10 & 0 & 50 \end{pmatrix}$$

1. Calculate the normal and shearing stress on the surface intersecting the point and parallel to the plane $2x_1 + x_2 - 3x_3 = 9$.

2. Calculate the invariants of the stress tensor and write down the characteristic equation for the principal values.

Exercise 7.1: For the stress state given by (arbitrary units),

$$[\sigma] = \begin{pmatrix} 100 & 20 & 0 \\ 20 & 60 & 0 \\ 0 & 0 & -50 \end{pmatrix}$$

Calculate

1. The principal stresses and orientations
2. Maximum shear stress
3. The octahedral shear stress

Exercise 7.2: For the two dimensional stress state (arbitrary units),

$$[\sigma] = \begin{pmatrix} 100 & 20 \\ 20 & 60 \end{pmatrix}$$

Draw the Mohr's cycle to evaluate the principal stresses and their orientations.

Exercise 8: For the stress state given by (arbitrary units),

$$[\sigma] = \begin{pmatrix} 56 & 0 & 0 \\ 0 & 35 & 0 \\ 0 & 0 & 14 \end{pmatrix}$$

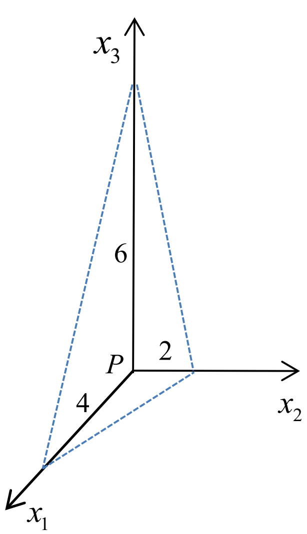
Calculate,

1. The maximum shear stress and its orientation with respect to the principal directions.
2. The shear stress on the octahedral plane.

Exercise 9: The stresses at a point are given with the following matrix (arbitrary units),

$$[\sigma] = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

Determine : (a) the principal stresses and (b) their directions (i.e. principal directions)



Exercise 10: The stresses at a point are given with the following matrix (arbitrary units),

$$[\sigma] = \begin{pmatrix} 7 & -5 & 0 \\ -5 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Determine the stress vector $\boldsymbol{t}$ at point P that belongs to a plane parallel to the ABC plane shown in the Figure (number on the axes indicate the coordinate of the corresponding point).

The following two problems were given last year in the Mid Term examination of the course

Problem 1: For the following state of stress, calculate the principal stresses and maximum shear stress,

$$\sigma_{11} = -5C; \quad \sigma_{22} = C; \quad \sigma_{33} = C$$
$$\sigma_{12} = -3C; \quad \sigma_{23} = \sigma_{31} = 0$$

Problem 2: For the following stress field, (1) what are the body forces to assure equilibrium?

(2) Calculate at point *P* (4, -4, 7) the stress vector on the sphere $x_1^2 + x_2^2 + x_3^2 = 81$ passing through *P*.

$$\sigma_{11} = -2x_1^2 + 3x_2^2 - 5x_3; \quad \sigma_{22} = -2x_2^2; \quad \sigma_{33} = 3x_1 + x_2 + 3x_3 - 5$$
$$\sigma_{12} = x_3 + 4x_1 x_2 - 7; \quad \sigma_{13} = -3x_1 + x_2 + 1; \quad \sigma_{23} = 0$$